Calling that mathematician... *(see cartoon on page 12)*

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The election is over. All 430 votes have all been tallied. But there is no winner, for the top three candidates all received the same number of votes. Enter preferential ballots.

In preferential balloting, each person votes for *all* candidates, indicating his/her order of preference of the candidates. Thus, with four candidates in the race, there are altogether $4! = 24$ possible ways a person can vote, reflecting the 24 possible orderings, or **preference schedules**, of Frumpf, Gluck, Ray, and Smiff. The results of the election can then be described by listing how many people voted for each of the 24 preference schedules. This is depicted in the cartoon on page 12, and, more fully, at the bottom of this page.

But how do you tell who won? Strange things can happen when you conduct preferential balloting without specifying the method of tabulation in advance -- and then you really need a mathematician!

There are a number of ways of tallying the ballots. In the **plurality method**, the winner is the candidate who received the largest number of first-place votes; preferences are in effect ignored, as is the case with standard elections where each person votes for just one candidate. In the **run-off method**, all but the two candidates A and B who received the most first-place votes are eliminated; the winner is the one of A and B who, accordingly to the preference schedules, would have received the most votes in a two-candidate race. In the **sequential run-off method**, the candidate with the least number of first place votes is eliminated, the preference schedules are retabulated and the process is repeated; when two candidates are left, the plurality method (or run-off method) is applied to determine the winner. In the **Borda count method** with four candidates, each candidate receives 3 points for each first-place vote, 2 points for each second-place vote, and 1 point for each third-place vote; the winner is the candidate who receives the greatest total count. In the **Condorcet method**, each candidate A is compared to each other candidate, and is assigned a number of points equal to the number of candidates that A would have defeated in a one-on-one election.

Your students can determine who would win the election by each of the methods above (answers are provided below) and then use preferential balloting for other situations, such as determining their favorite soft drink, rock star, or mathematician.

What is striking is that it is possible for these methods to lead to different winners. A situation where the methods give four different winners is featured in the videotapes accompanying *For All Practical Purposes*. Explanations and further information about preferential ballots can be found in both books reviewed on page 10. ■

F	G	R	S	F	G	R	S
G	F	F	G	G	F	G	F
R	R	G	R	S	S	F	G
S	S	S	F	R	R	S	R
14	22	16	31	11	9	27	17

Left Wall

F	G	R	S	F	G	R	S
R	R	F	F	R	R	G	G
G	S	S	R	S	F	F	F
S	F	G	G	G	S	S	R
6	11	37	38	36	22	7	3

Front Wall

F	G	R	S	F	G	R	S
S	S	S	R	S	S	S	R
G	F	F	G	R	R	G	F
R	R	G	F	G	F	F	G
10	25	7	19	33	21	6	2

Right Wall

Answers: Neither the **plurality method** nor the **run-off method** determines a winner, since three candidates are tied for first place. Using the **sequential run-off method**, Ray is eliminated first, and his name is deleted from each of the preference schedules -- for example, the 16 votes for preference schedule R-F-G-S now become 16 votes for F-G-S ; Smiff is eliminated next, and finally, we see that Frumpf defeats Gluck. Using the **Borda count method**, we find that Frumpf receives 694 points, Smiff 645 points, Ray 641 points, and Gluck 590 points, so that this method also gives the nod to Frumpf. Finally, using the **Condorcet method**, we find that F defeats G by 227-203, R by 224-206, and S by 243-188, so that Frumpf defeats all three other candidates in one-on-one elections and receives the maximum of 3 points. On the whole, a convincing win for Frumpf. ■

Spreading the Word... *(Continued from page 2)*

many teachers are integrating the materials into existing courses. It is a very exciting time to be a mathematics teacher with all the changes going on in structure, content, and pedagogy. Using discrete math is a way of being involved in all three.

Your newsletter is wonderful! Knowing that there's a whole network of others around the country, I no longer feel like an isolated disciple of discrete math.